

Application of the multi-objective optimization method for designing a powered Stirling heat engine: Design with maximized power, thermal efficiency and minimized pressure loss



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ABSTRACT

In the recent years, numerous studies have been done on Stirling cycle and Stirling engine which have been resulted in different output power and engine thermal efficiency analyses. Finite speed thermodynamic analysis is one of the most prominent ways which considers external irreversibilities. In the present study, output power and engine thermal efficiency are optimized and total pressure losses are minimized using NSGA algorithm and finite speed thermodynamic analysis. The results are successfully verified against experimental data.

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1. Introduction

One of the greatest issues for the societies in the near future is security of sustainable energy [1]. Great amount of enterprise will be vital to satisfy future energy requirements with climate-friendly energy technologies [2–5], comprises electricity produced from solar and wind energy [6–8], bio-fuels [9,10], and carbon dioxide (CO₂) capture and storage (CCS) [11–14]. In this regard, scrutinized planning with respect to the prospect plays an important role toward better future.

Carbon dioxide emission is one of the restrictions of applying fossil fuels in all over the world which motivates expansion of higher efficiency and maximal electric energy per 1 kg CO₂ emitted technologies [15,16]. Use of Stirling cycles can lead to reduce the CO₂ emissions to the atmosphere.

Stirling cycle is one of the main primitive standard air cycles for heat engines [17,18]. External heat source and high efficiency are the Stirling engines advantages. Stirling engines can use solar energy which is available in one-third of the day, in the result solar/fuel hybrids are recommended. The combustion of the Stirling engine is continuous process and can burn all types of fuel with any quality [18–20]. The Stirling engine can theoretically be a highly efficient engine to convert heat into mechanical work at the Carnot efficiency when the ideal regeneration, isothermal compression and expansion are considered. The thermal limit for the operation of a Stirling engine depends on the working temperatures of the heater and cooler sides. In most instances, the engine operates with a heater and cooler temperature of 923 and 338 K, respectively [21]. The engine efficiency varies from about 30 to 40% which results in forming a typical temperature range of 923–1073 K, and normal operating speed range from 2000 to 4000 rpm [22–27]. The influence of the regenerator effectiveness, the dead volume on the power output and the thermal efficiency are studied by Kongtragool and Wongwises [24].

Carlson et al. provided a non-isothermal heat exchanger model which eliminates the necessity for infinite heat transfer time correspond to slow engine speed that is impractical [28].

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Organ studied the effects of various parameters such as diameter, length and materials on regenerator performance, irreversibilities and temperature gradient in Stirling engine regenerator while the regenerator is optimized [29,30].

Martaj et al. presented a thermodynamic analysis of a low temperature differential Stirling engine at steady state operation, and energy, entropy and exergy balances were presented at each main element of the engine [31].

Formosa and Despesse modeled engine power output and efficiency due to dead volume by implementing of the isotherm model [32].

Timoumi et al. developed a numerical model based on lumped analysis approach which takes into account losses and is implemented for optimization of GPU-3 Stirling engine [33–35].

The effects of heat transfer, regeneration time, and imperfect regeneration on the performance of the irreversible Stirling engine cycle are investigated by Wu et al. [36]. Li et al. developed a mathematical model for the overall thermal efficiency of solar powered high temperature differential dish Stirling engine with finite heat transfer and the irreversibility of regenerator and optimized the absorber temperature and corresponding thermal efficiency [37]. Tlili studied the effects of regenerating effectiveness and heat capacitance rate of external fluids in heat source/sink at maximum power and efficiency [38]. The effects of addressed irreversibilities of regeneration and heat transfer of heat/sink sources were investigated by Kaushik and Kumar [39,40].

This paper concentrates on the evaluation of the losses because of the irreversibilities in the Stirling cycle which is an issue of substantial interest to those concerned with the performance and analysis of heat engines. Our researches and latest studies by others [41,42] have scrutinized that irreversibilities in the thermodynamic cycle have a specific significance in forecasting the performance of Stirling engines. Remarkable attempts have been made over the past years to completely recognize the irreversibilities associated losses [43,44]. Finite Speed of the piston, Friction and Throttling in the regenerator cause the internal irreversibilities which have to be taken into account for validating of any design of solar Stirling engines computation. One of the earliest efforts for a validation of the solar Stirling engines scheme of computation has been done by Costea et al. [45]. More exactly validation has been possible to accomplish through a validation of the schemes of computation for as many as possible Stirling engines. Probably, it can be done by using the new branch of irreversible thermodynamics, so-called “Thermodynamics with Finite Speed, and the Direct Method” [46].

Based on this, in 2002 it became possible to obtain the most powerful validation of a scheme of computation of the performances (efficiency and power) for 12 Stirling engines (the most performing in the world) working in 16 regimes [47]. Only with such a powerful “tool” it is possible to obtain a validation for a scheme of computation of solar Stirling engines performances for 5 solar Stirling engines [48–50].

This method has been employed in a number of models for the analysis and optimization of Stirling engines including the impact of irreversibilities in the cycle, successfully. However, analyses that are based on the entropy generation or exergy techniques do not relate the irreversibilities to the physical phenomena that cause them. The model presented here directly associates the irreversibilities to the operation of the cycle at finite speed. On the basis of the entropy generation techniques, for the studying of Stirling engine cycle performance, Costea et al. [48,49] have included the impacts of heat transfers, imperfect heat regeneration and irreversibilities of the cycle including pressure loss associated with the finite speed of the piston and Displacer throughout processes as gas passes through the regenerator,

heater and cooler, and finally mechanical friction due to the motion of the moving parts.

For considering all mentioned issues, we have studied three objective functions: output power, the thermal efficiency of the entire solar Stirling system and pressure losses. In Addition, the multi-objective optimization is conducted with eleven decision variables including the temperatures of heat source and heat sink and their difference with working fluids, rotation speed, mean effective pressure and stroke.

Solution of the multi-objective optimization problems is an extremely difficult goal which requires the simultaneous satisfaction of a number of different and even conflicting objectives. Evolutionary algorithms (EA) were initially extended and employed during the mid-eighties in an attempt to stochastically solve problems of this generic class [51]. A reasonable solution to a multi-objective problem is to investigate a set of solutions, each of which satisfies the objectives at an acceptable level without being dominated by any other solution [52]. Multi-objective optimization problems, in general, show a possibly uncountable set of solutions namely as Pareto frontier, whose evaluated vectors represent the best possible trade-offs in the objective function space. In this term, multi-objective optimization of different thermodynamic and energy systems has been paid attention by researchers nowadays [53–58].

2. Stirling system

As shown in Fig. 1, Stirling cycle consists of four major processes. Process 1–2 is an isothermal process, in which the compressing working fluid rejects the heat at constant temperature (T_c) to heat sink which has a constant temperature (T_L). Then the working fluid crosses over the regenerator and is warmed up to T_h in an isochoric process 2–3. In process 3–4, the working fluid expands at a constant temperature, T_h , and obtains the heat from the heat source at a constant temperature (T_H). Last process (4–1), is an isochoric cooling process, where the regenerator absorbs heat from the working fluid. In an actual cycle it is impractical to have an ideal heat transfer in the regenerator in which the entire amount of absorbed heat (in the process 4–1) is transferred to the working fluid into the isochoric heating process (process 2–3).

3. Analysis of the Stirling engine cycle with irreversibilities

The pressure losses presented in this task is as follows [45–49]:

$$\sum \Delta p_i = \Delta p_{throt_r} + \Delta p_f + \Delta p_w \quad (1)$$

where Δp_{throt_r} is the pressure drop coming of the internal friction of the current which takes place in the regenerator and is insignificant in heat exchangers (coolers and heaters) [45–50].

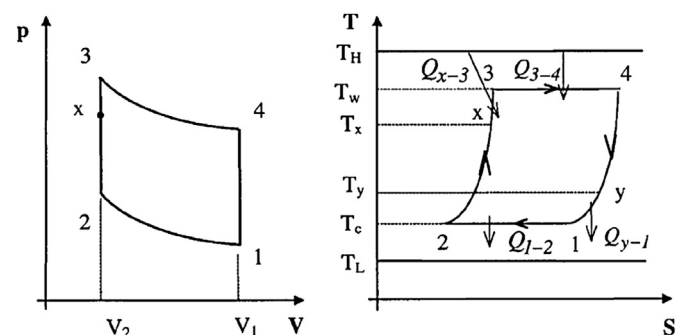


Fig. 1. P - V and T - S diagrams of an isothermal Stirling engine cycle [48].

$$\Delta p_{throt_R} = \frac{15}{\gamma} \left[\frac{p_m}{2R(\tau+1)(T_L + \Delta T_L)} \left(\frac{s^2 n_r^2}{900} \right) \right] \cdot N \cdot \left(\frac{D_c^2}{N_R D_R^2} \right)^2 \quad (1a)$$

Δp_f is the pressure drop which comes of the mechanical friction of the parts of the engine and has been experimentally [45–49] obtained:

$$\Delta p_f = \frac{(0.94 + 0.0015s n_r) \cdot 10^5}{3\mu'} \left(1 - \frac{1}{\lambda} \right) \quad (1b)$$

$$\mu' = 1 - \frac{1}{3\lambda} \quad (1c)$$

And finally the pressure drop coming of the piston speed [45–49] is calculated as follows:

$$\Delta p_w = \left(\frac{s n_r}{60} \right) \cdot \left(\frac{4p_m}{(1+\lambda)(1+\tau)} \right) \left(\frac{\lambda \cdot \ln \lambda}{\lambda - 1} \right) \sqrt{\frac{\gamma}{R}} \left(\frac{1}{\sqrt{T_L + \Delta T_L}} \right) \left[1 + \sqrt{\frac{T_H - \Delta T_H}{T_L + \Delta T_L}} \right] \quad (1d)$$

ΔQ_R is the heat loss during the two regenerative processes in the cycle which can be obtained from Eq. (2) [45–47]:

$$\Delta Q_R = m_g C_{v_g} X (T_H - \Delta T_H - T_L - \Delta T_L) \quad (2)$$

The heat released between heat source and working fluid (Q_h) is obtained as follows [45–47]:

$$Q_h = m_g \left(1 - \frac{\Delta p_w \cdot (\lambda + 1)(\tau + 1)}{4p_m} - \frac{b \Delta p_{throt_R}}{2p_m} - \frac{f \Delta p_f}{p_m} \right) R(T_H - \Delta T_H) \ln \lambda \quad (3)$$

The net heat released from the heat source (Q_H) is obtained as follows [45–47]:

$$Q_H = Q_h + \Delta Q_R \quad (4)$$

The pure flux transferred to working fluid is obtained as follows [45–47]:

$$\dot{Q}_H = (Q_h + \Delta Q_R) \frac{n_r}{60} \quad (5)$$

Substituting Eqs. (2)–(4) into Eq. (5), the pure flux transferred to working fluid, we have [46–52]:

$$\dot{Q}_H = \left[m_g \left(1 - \frac{\Delta p_w \cdot (\lambda + 1)(\tau + 1)}{4p_m} - \frac{b \Delta p_{throt_R}}{2p_m} - \frac{f \Delta p_f}{p_m} \right) R(T_H - \Delta T_H) \ln \lambda + X \cdot m_g C_{v_g} (T_H - \Delta T_H - T_L - \Delta T_L) \right] \cdot \frac{n_r}{60} \quad (6)$$

The output power of the engine is calculated as follows [45]:

$$Power = \eta \dot{Q}_H = \left(1 - \frac{T_L + \Delta T_L}{T_H - \Delta T_H} \right) \cdot \eta_{II,irr} \dot{Q}_H \quad (7)$$

$$\Delta T_L = T_c - T_L \quad (7a)$$

$$\Delta T_H = T_H - T_h \quad (7b)$$

$$\eta_c = \left(1 - \frac{T_L + \Delta T_L}{T_H - \Delta T_H} \right) \quad (7c)$$

By placing Eqs. (7a)–(7c) in Eq. (7), we have:

$$\eta = \eta_c \eta_{II,irr} \quad (8a)$$

$$Power = \eta_c \eta_{II,irr} \dot{Q}_H \quad (8b)$$

where η_c is Carnot efficiency and $\eta_{II,irr}$ is the efficiency of the second law. In this evaluation, each one of the irreversibilities has an independent effect which is taken into consideration in the efficiency of the second law, in such a way that [45–50]:

$$\eta_{II,irr} = \eta_{II,irr(X)} \cdot \eta_{II,irr(\Delta p)} \quad (9)$$

The effect of imperfect regenerating is calculated in $\eta_{II,irr(X)}$ which the following are [45–49]:

$$\eta_{II,irr(X)} = \frac{1}{1 + \left(\frac{X}{(\gamma - 1) \ln \lambda} \right) \cdot \eta_c} \quad (10)$$

with

$$X = yX_1 + (1 - y)X_2 \quad (11)$$

where y is the second adjusting coefficient of the method, and X_1, X_2 are regenerative losses coefficient values corresponding to an “optimistic”, respectively “pessimistic” evaluation of them. The value of y has been determined from experimental data of 4 solar Stirling engines [47]. Setting $y = 0.72$ provided the best fit between analytical results and experimental one, so this value is carried out as an adjusting coefficient in the theoretical analysis. In the above relation [46–49]:

$$X_1 = \frac{1 + 2M + e^{-B}}{2(1 + M)} \quad (11a)$$

$$X_2 = \frac{M + e^{-B}}{1 + M} \quad (11b)$$

$$M = \frac{m_g C_{v_g}}{m_R C_R} \quad (11c)$$

$$m_R = \frac{\pi^2 D_R^2 L d \rho_{st}}{16(b + d)} \quad (11d)$$

$$B = (1 + M) \cdot \frac{h A_R}{m_g C_{v_g}} \cdot \frac{30}{n_r} \quad (11e)$$

In which the convection coefficient h is computed as [46–49]:

$$h = \frac{0.395 \left(\frac{4p_m}{RT_L} \right) \cdot \left(\frac{s \cdot n_r}{30} \right)^{0.424} C_p v^{0.576}}{(1 + \tau) \left[1 - \frac{\pi}{4 \left[\left(\frac{b}{d} \right) + 1 \right]} \right]} \cdot D_R^{0.576} Pr^{0.667} \quad (12)$$

On the other hands [47]:

$$A_R = \frac{\pi^2 D_R^2 L}{4(b + d)} \quad (13)$$

The effect of mechanical friction, the speed of the piston and the pressure drop in regenerator are calculated in $\eta_{II,irr(\Delta p)}$ [45–49]:

$$\eta_{II,irr(\Delta p)} = 1 - \frac{3\mu' \cdot \sum \frac{\Delta p_i}{p_1}}{\eta' \cdot \left(\frac{T_H - \Delta T_H}{T_L + \Delta T_L} \right) \cdot \ln \lambda} \quad (14)$$

$$\eta' = \eta_c \cdot \eta_{II,irr(1-\epsilon_R)} \quad (14a)$$

$$p_1 = \frac{4p_m}{(1+\lambda)(1+\tau)} \quad (14b)$$

The above analysis shows that the pressure losses and their effect on efficiency and output power of the engine depend on the mean piston speed and hence the engine rotation speed.

4. Multi-objective optimization with evolutionary algorithms

4.1. General concepts of the multi-objective optimization

Consider a decision-maker who wants to optimize K objectives that objectives are non-commensurable and the decision-maker has no clear priority of each objective over the others. Without loss of generality, all objectives are of the minimization type. For a minimization type objective, it can be renewed to the maximization type by multiplying negative one. A minimization multi-objective decision problem with K objectives is defined as follows:

Given a non-dimensional decision variable vector $x = \{x_1, \dots, x_n\}$ in the solution space X find a vector x^* that minimizes a given set of K objective functions $f(x) = \{f_1(x^*), \dots, f_k(x^*)\}$. The solution space X is generally restricted by a series of constraints, such as $g_j(x^*) = b_j$ for $j = 1, \dots, m$, and bounds on the decision variables [52]. Overall, no solution vector X exists that minimizes all the K objective functions simultaneously. In another word, in various real-life problems, objectives at issues conflict each other. So optimizing X with respect to a uni-objective often leads to unacceptable results with respect to the other objectives. Thus a new concept, well-known as the “Pareto optimum solution”, is implemented in multi-objective optimization problems. A feasible solution X is called “Pareto optimal” if there is no other feasible solution Y that dominates solution X . By definition, a feasible solution Y is said to dominate another feasible solution X , if and only if, $f_i(Y) \leq f_i(X)$ for $i = 1, \dots, k$ and $f_j(Y) < f_j(X)$ for at least one objective function. This means that a feasible vector X is called Pareto optimal if there is no other feasible solution Y that would reduce some objective function without causing a simultaneous increase in at least one other objective function. The set of all feasible non-dominated solutions in X is referred to as the “Pareto optimal set”, and for a given Pareto optimal set, the corresponding objective function values in the objective space are called the “Pareto optimal frontier” [52].

Notice Fig. 2 for having an effective insight into the concept of Pareto optimal. The objective function space is illustrated for an optimization problem with two objective functions f_1 and f_2 . The feasible and infeasible areas are shown as the areas inside and outside of the curve respectively. Each point in the feasible area can be represented as a solution of the problem. In Fig. 2, the values of both functions f_1 and f_2 . For point M is lower than the corresponding values of the point J . Thus the point M dominates point J or point M is better than a point J . In the same way points L and N dominate M . But points like I and K neither dominate M nor are dominated by M . Therefore, only those points that are located in the left-down parts of M will dominate M (like L and N). Now consider point R , there is no point in the feasible area located in the left-

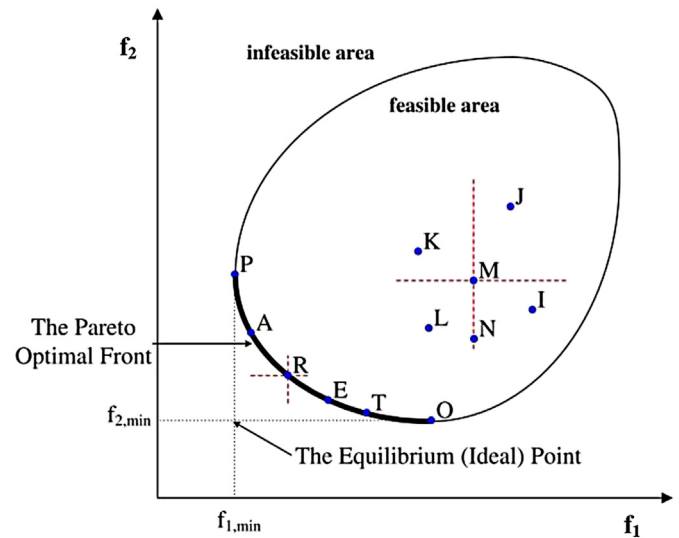


Fig. 2. Schematic of the objectives space.

down part of it. Thus R is not dominated by any other feasible point. Therefore R is a Pareto optimal. This is true for points P, A, R, E, T, O and all of the other points located at the bold curve indicated as “The Pareto Optimal Frontier” in Fig. 2. Notice that, in the feasible area, the minimum values of f_1 and f_2 belong to points P and O respectively. Thus P and O are the solutions for uni-objective optimization problems which their objective functions are f_1 and f_2 , respectively. Other points on the Pareto frontier are also optimal, but each of them should be chosen as the final solution which is required a decision-making process.

4.2. Optimization via EA

In the present study, the Pareto frontier is achieved using the Genetic Algorithm (GA) known as a tributary of evolutionary algorithm. Genetic Algorithms were evolved by John Holland in the 1960s for involving the mechanisms of natural adaptation into numerical optimization and computer algorithms [54]. They are employed as a computer simulation in which a population of abstract representations (called chromosomes or the genotype of the genome) of candidate solutions (called individuals, creatures, or phenotypes) to an optimization problem develops toward better solutions. The progress often begins from a population of randomly generated individuals and happens in generations which the fitness of every individual in the population is evaluated. In each generation; multiple individuals are stochastically selected from the existing population (based on the fitness), and modified (recombined and possibly randomly changed) to create a new population. The new population is applied in the next iteration. Generally, the algorithm ends when either a satisfactory fitness level has been overtaken for the population, or a maximum number of generations have been made. If the algorithm has finished owing to a maximum number of generations, a satisfactory solution may or may not be found. In genetic algorithms, a candidate solution to an issue is routinely entitled a chromosome, and the evolutionary livability of every chromosome is represented by a fitness function which is a powerful optimization method for nonlinear problems [53,55].

Furthermore, multi-objective evolutionary algorithms (MOEAs) have been expanded over the recent decades by many tests on twisted mathematical problems and on real-world engineering subjects, they can remove the obstacles of classical procedures

[53,55]. The structure of the MOEA employed in the present investigation by the authors is shown in Fig. 3 [54]. The real values of decision variables are taken into account instead of their binary coded.

4.3. Objective functions, decision variables and constraints

Three objective functions for this study are the system pressure losses ($\sum \Delta P_i$), the output power (*Power*) and the Stirling engine thermal efficiency (η), respectively.

In this paper eleven decision variables have been considered as follows:

- n_r : Engine's rotation speed
- P_m : Mean effective pressure
- s : Stroke
- T_H : Temperature of heat source
- T_L : Temperature of heat sink
- ΔT_L : Temperature difference between heat source and working fluid
- ΔT_H : Temperature difference between heat sink and working fluid
- N_R : Number of gauzes of the matrix
- D_C : Piston Diameter
- D_R : Regenerator Diameter
- L : Regenerator's length

The objective functions with respect to following constraints have been solved:

$$1200 \leq n_r \leq 3000 \text{ rpm} \quad (15)$$

$$0.69 \leq P_m \leq 6.89 \text{ MPa} \quad (16)$$

$$0.06 \leq s \leq 0.1 \text{ m} \quad (17)$$

$$800 \leq T_H \leq 1300 \text{ K} \quad (18)$$

$$288 \leq T_L \leq 360 \text{ K} \quad (19)$$

$$64.2 \leq \Delta T_H \leq 237.6 \text{ K} \quad (20)$$

$$5 \leq \Delta T_L \leq 25 \text{ K} \quad (21)$$

$$250 \leq N_R \leq 400 \quad (22)$$

$$6 \times 10^{-3} \leq L \leq 73 \times 10^{-3} \text{ m} \quad (23)$$

$$0.05 \leq D_C \leq 0.14 \text{ m} \quad (24)$$

$$0.02 \leq D_R \leq 0.06 \text{ m} \quad (25)$$

5. Decision-making in the multi-objective optimization

A procedure of decision-making is required in multi-objective optimization to choose the final optimal solution from available solutions. Great numbers of techniques for decision-making process in decision problems exist. The methods can be utilized in decision-making to opt a final optimal solution from the Pareto frontier which is gained through optimization. Insomuch dimension of various objectives in a multi-objective optimization issue might be various (for example in this work the thermal efficiency objective has no dimension while the dimension of the output power is in Watt). It is true that, dimensions and scales of objectives space have to be unified at first. In turn, objectives vectors have to be non-dimensionalized before the decision-making procedure. There are some ways of non-dimensionalization utilized in decision-making including linear, Euclidian, and fuzzy.

• Linear non-dimensionalization

Consider the matrix of objectives at various points of the Pareto frontier is denoted by F_{ij} where i is index for each point on the Pareto frontier and j is the index for each objective in objectives space. Therefore a non-dimensionalized objective, F_{ij}^n , is defined as,

$$F_{ij}^n = \frac{F_{ij}}{\max(F_{ij})} \text{ For maximizing objective} \quad (26a)$$

$$F_{ij}^n = \frac{F_{ij}}{\max(1/F_{ij})} \text{ For minimalizing objective} \quad (26b)$$

• Euclidian non-dimensionalization

In this method, a non-dimensionalized objective, F_{ij}^n , is defined as,

$$F_{ij}^n = \frac{F_{ij}}{\sqrt{\sum_{i=1}^m (F_{ij})^2}} \text{ For minimizing and maximizing objectives} \quad (27)$$

• Fuzzy non-dimensionalization

In this method, a non-dimensionalized objective, F_{ij}^n , is defined as [55,56],

$$F_{ij}^n = \frac{F_{ij} - \min(F_{ij})}{\max(F_{ij}) - \min(F_{ij})} \text{ For maximizing objectives} \quad (28a)$$

$$F_{ij}^n = \frac{\max(F_{ij}) - F_{ij}}{\max(F_{ij}) - \min(F_{ij})} \text{ For minimizing objectives} \quad (28b)$$

In this work, most recognized and common types of decision-making processes including the fuzzy Bellman–Zadeh, LINMAP and TOPSIS methods are implemented in parallel and the final optimal solution has been decided based on engineering experience and criteria among solutions which is suggested by these three methods. The Bellman–Zadeh method utilizes the fuzzy non-dimensionalization while the other methods (LINMAP and TOPSIS) employ Euclidian non-dimensionalization. The following sections are presented here in order to describe these decision-making algorithms.

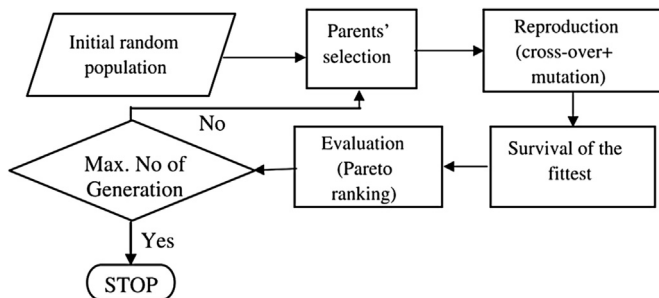


Fig. 3. Scheme for the multi-objective evolutionary algorithm used in the present study.

5.1. Bellman–Zadeh decision-making method

When using the Bellman–Zadeh approach, each $F_j(X)$ is replaced by a fuzzy objective function or a fuzzy set,

$$A_j = \{X, \mu_{A_j}(X)\} \quad X \in L, j = 1, 2, \dots, k \quad (29)$$

where $\mu_{A_j}(X)$ is a membership function of A_j [59].

A final decision is defined by the Bellman and Zadeh model as the intersection of all fuzzy criteria and constraints and is represented by its membership function. A fuzzy solution D with setting up the fuzzy sets (Eq. (30)) is turned out as a result of the intersection $D = \cap_{j=1}^k A_j$ with a membership function,

$$\mu_D(X) = \cap_{j=1}^k \mu_{A_j}(X) = \min_{j=1, \dots, k} \mu_{A_j}(X), \quad X \in L \quad (30)$$

Using Eq. (28a), it is possible to obtain the solution proving the maximum degree as follows,

$$\max \mu_D(X) = \max_{X \in L} \min_{j=1, \dots, k} \mu_{A_j}(X) \quad (31)$$

$$X^0 = \arg \max_{X \in L} \min_{j=1, \dots, k} \mu_{A_j}(X) \quad (32)$$

To obtain Eq. (31), it is necessary to build membership functions $\mu_{A_j}(X)$; $j = 1, \dots, k$, reflecting a degree of achieving “own” optimal by the corresponding $F_j(X)$, $X \in L$; $j = 1, \dots, k$. This is satisfied by the implement of the membership functions [60]. The membership function of objectives and constraints, linear or nonlinear, can be chosen depending on the context of problem. One of possible fuzzy convolution schemes is presented below [61].

Initial approximation for X -vector is chosen. Maximum (minimum) values for each criterion $F_j(X)$ are established via scalar maximization (minimization). Results are denoted as “ideal” points $\{X_j^0, j = 1, \dots, m\}$.

The matrix table $\{T\}$, where the diagonal elements are “ideal” points, is defined as follows:

$$\{T\} = \begin{bmatrix} F_1(X_1^0) & F_2(X_1^0) & \dots & F_n(X_1^0) \\ F_1(X_2^0) & F_2(X_2^0) & \dots & F_n(X_2^0) \\ \vdots & \vdots & \ddots & \vdots \\ F_1(X_n^0) & F_2(X_n^0) & \dots & F_n(X_n^0) \end{bmatrix} \quad (33)$$

Maximum and minimum bounds for the criteria are defined:

$$F_i^{\min} = \min_j F_j(X_j^0), \quad i = 1, \dots, n \quad (34a)$$

$$F_i^{\max} = \max_j F_j(X_j^0), \quad i = 1, \dots, n \quad (34b)$$

The membership functions are assumed for all fuzzy goals as follows.

For minimized objective functions:

$$\mu_{F_i}(X) = \begin{cases} 0 & \text{if } F_i(X) > F_i^{\max}, \\ \frac{F_i^{\max} - F_i}{F_i^{\max} - F_i^{\min}} & \text{if } F_i^{\min} < F_i \leq F_i^{\max}, \\ 1 & \text{if } F_i(X) \leq F_i^{\min} \end{cases} \quad (35a)$$

For maximized objective functions:

$$\mu_{F_i}(X) = \begin{cases} 1 & \text{if } F_i(X) > F_i^{\max}, \\ \frac{F_i - F_i^{\min}}{F_i^{\max} - F_i^{\min}} & \text{if } F_i^{\min} < F_i \leq F_i^{\max}, \\ 0 & \text{if } F_i(X) \leq F_i^{\min} \end{cases} \quad (35b)$$

Fuzzy constraints are formulated:

$$G_j(X) \leq G_j^{\max} + d_j, \quad j = 1, 2, \dots, k \quad (36)$$

where d_j is a subjective parameter that denotes a distance of admissible displacement for the bound $G_j^{\max}$ of the j th constraint. Corresponding membership functions are defined in following manner [55,56]:

$$\mu_{G_i}(X) = \begin{cases} 0 & \text{if } G_i(X) > G_i^{\max} \\ 1 - \frac{G_j(X) - G_j^{\max}}{d_j} & \text{if } G_i^{\max} < G_i(X) \leq G_i^{\max} + d_j \\ 1 & \text{if } G_i(X) \leq G_i^{\max} \end{cases} \quad (37)$$

A final decision is determined as the intersection of all fuzzy criteria and constraints represented by its membership functions. This problem is reduced to the standard nonlinear programming problems: to find such values of X and k that maximize k subject to

$$\begin{aligned} \lambda &\leq \mu_{F_i}, \quad i = 1, 2, \dots, n \\ \lambda &\leq \mu_{G_j}, \quad j = 1, 2, \dots, k \end{aligned} \quad (38)$$

The solution of the multi-criteria problem discloses the meaning of the optimality operator and depends on the decision-makers experience and problem understanding.

5.2. LINMAP decision-making method

An ideal point on the Pareto frontier is the point in which each objective is optimized regardless to satisfaction of other objectives. It is clear that in the multi-objective optimization it is impossible to have each objective in its optimal condition which can be obtained only in a single-objective optimization. Therefore, the ideal point is not located on the Pareto frontier. In the LINMAP method, after Euclidian non-dimensionalization of all objectives, the distance of each solution on the Pareto frontier from the ideal point denoted by d_{i+} is determined as follow,

$$d_{i+} = \sqrt{\sum_{j=1}^n F_{ij} - F_j^{\text{ideal}}} \quad (39)$$

where n denotes the number of objective while stands for each solution on the Pareto frontier ($i = 1, 2, \dots, m$). In Eq. (39), F_j^{ideal} is the ideal value for j th objective obtained in a single-objective optimization. In LINMAP method, the solution with minimum distance from ideal point is selected as a final desired optimal solution, hence, i index for a final solution, i_{final} is,

$$i_{final} = i \in \min (d_{i+}); i = 1, 2, \dots, m \quad (40)$$

5.3. TOPSIS decision-making method

In this method beside the ideal point a non-ideal point is defined. The non-ideal point is the ordinate in objectives space in which each objective has its worst value. Therefore, beside the solution distance from ideal point, d_{i+} , the solution distance from non-ideal point denoted by d_{i-} is utilized as a criterion for selection of the final solution. Hence [55,56],

$$d_{i-} = \sqrt{\sum (F_{ij} - F_j^{non-ideal})^2} \quad (41)$$

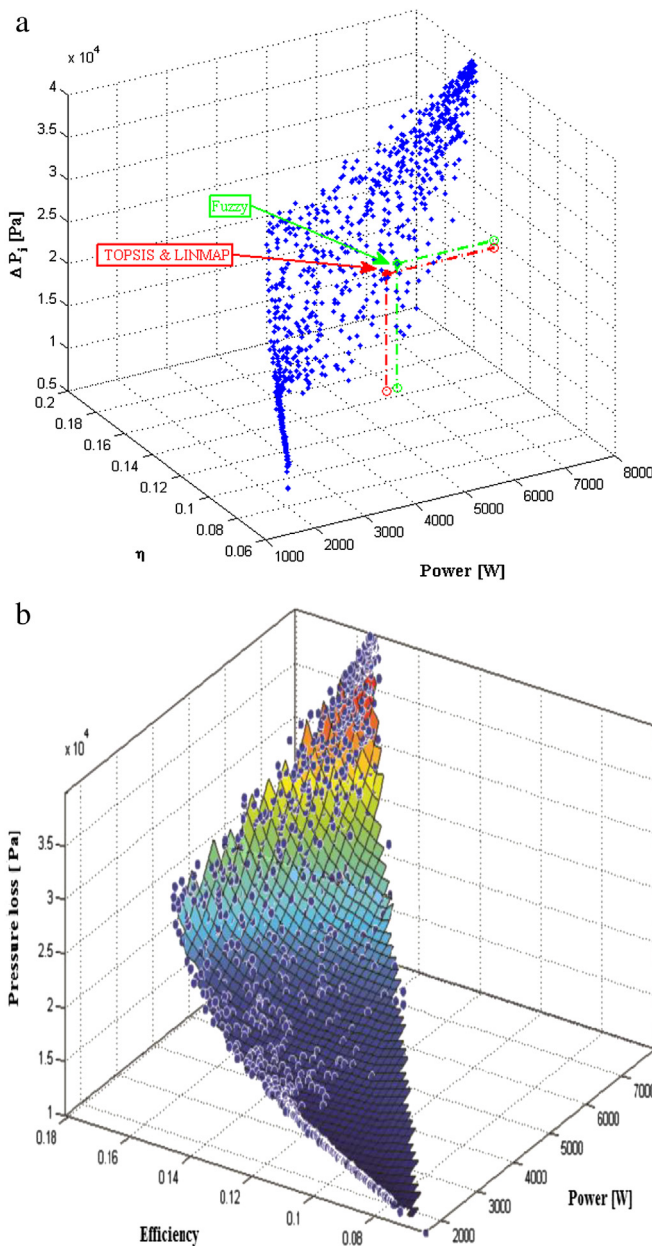


Fig. 4. (a) Pareto optimal frontier in objectives' space. (b) Pareto optimal frontier in objectives' space.

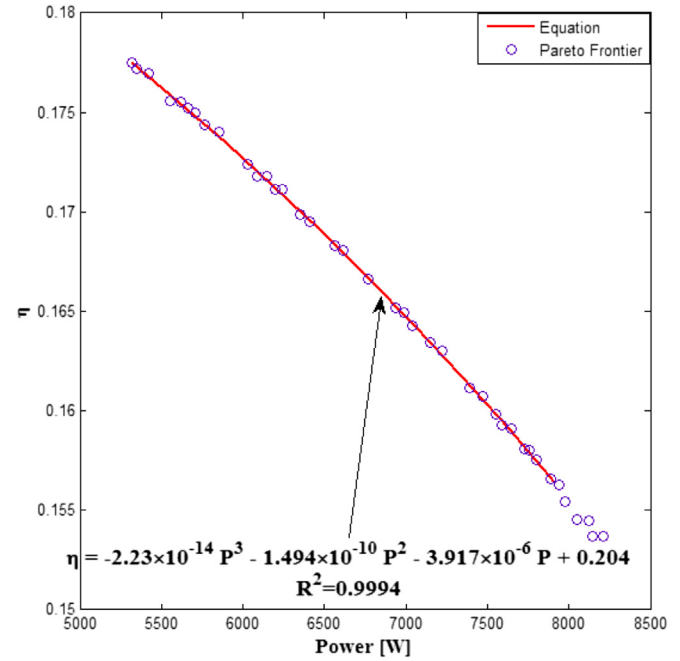


Fig. 5. Pareto optimal frontier in objectives' space (η – Power).

In continuing the TOPSIS method a Cl_i parameter is defined as follows,

$$Cl_i = \frac{d_{i-}}{d_{i+} + d_{i-}} \quad (42)$$

In TOPSIS method a solution with minimum Cl_i is selected as a desired final solution, therefore, if i_{final} is index for the final selected solution, we have,

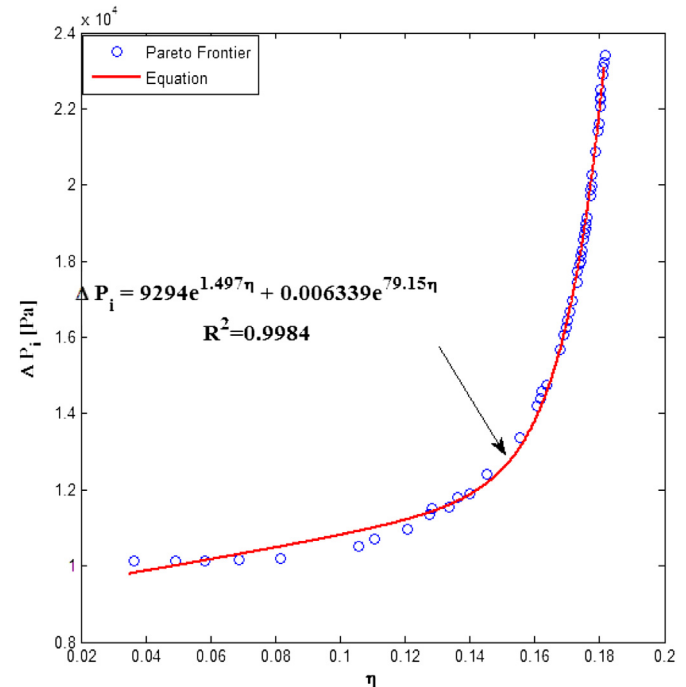


Fig. 6. Pareto optimal frontier in objectives' space ($\sum \Delta p_i$ – η).

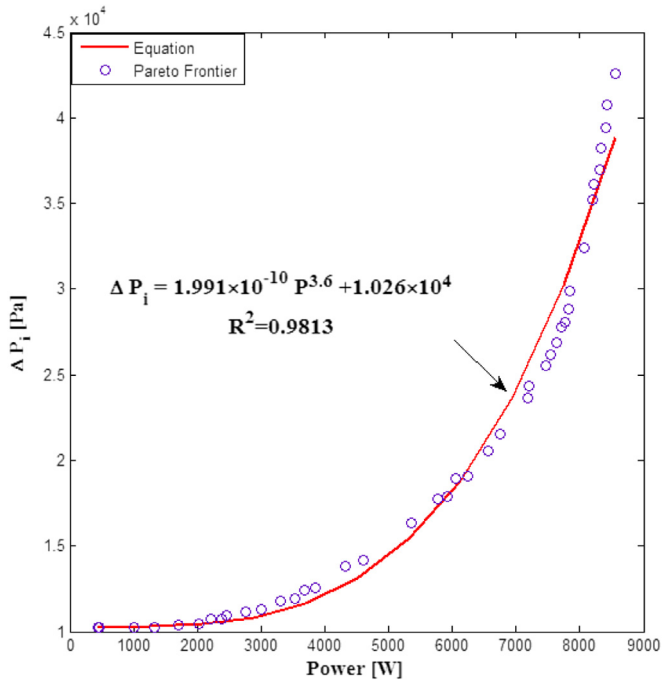


Fig. 7. Pareto optimal frontier in objectives' space ($\Sigma\Delta P_i$ – Power).

$$i_{final} = i \in \max(C_i); i = 1, 2, \dots, m \quad (43)$$

6. Results and discussion

The output power and thermal efficiency of the Stirling engine are maximized simultaneously and the pressure losses of the Stirling engine are minimized simultaneously using the multi-objective optimization method which works based on the NSGAII algorithm.

In this regard, optimization is performed with objective functions that are expressed by Eqs. (1), (7) and (8a) constraints which are expressed with Eqs. (15)–(25).

Design parameters (decision variables) of optimization are the engine's rotation speed, Mean effective pressure, Stroke, Number of gauzes of the matrix, Piston Diameter, Regenerator Diameter, regenerator length, temperature of the heat source, temperature of the heat sink, Temperature difference between the heat source and working fluid, Temperature difference between heat sink and working fluid.

In order to have consistency with previous works, specifications of the Stirling engine are considered as follows [35]:

$N = 8$, $v = 3.249 \times 10^{-5} \text{ (m}^2 \text{ s}^{-1}\text{)}$, $\gamma = 1.667$, $C_v = 3115.6 \text{ (J kg}^{-1} \text{ K}^{-1}\text{)}$, $Pr = 0.71$, $m_g = 0.001135 \text{ (kg)}$, $\rho_{st} = 8030 \text{ (kg m}^{-3}\text{)}$, $C_{pg} = 5193 \text{ (J kg}^{-1} \text{ K}^{-1}\text{)}$, $C_R = 502.48 \text{ (J kg}^{-1} \text{ K}^{-1}\text{)}$, $\lambda = 1.2$, $b = 6.88 \times 10^{-5}$, $f = 0.556$, $d = 4 \times 10^{-5} \text{ (m)}$.

Table 1
Decision-making of multi-objective optimal solutions.

Optimal solution	Decision variables										Objective functions			
	n_r [rpm]	P_m [kPa]	s [mm]	T_H [K]	T_L [K]	ΔT_H [K]	ΔT_L [K]	N_R	L [mm]	D_c [mm]	D_R [mm]	Power [kW]	η_{SE} [%]	$\Sigma\Delta P_i$ [kPa]
TOPSIS	2120	2550.3	60.5	989.6	298.4	74.4	11.8	339	70.0	101.6	59.5	6.076	14.56	19.690
LINMAP	2120	2550.3	60.5	989.6	298.4	74.4	11.8	339	70.0	101.6	59.5	6.076	14.56	19.690
Bellman-Zadeh	2056	2437.0	60.5	989.3	299.5	76.4	12.1	338	71.8	106.1	58.9	5.840	14.51	18.829
Experimental results [46]	2500	4130	100	977	288	–	–	308	22.6	22.6	69.9	3.96	12.7	–

Table 2

Error analysis based on the mean absolute percent error (MAPE) method.

Decision-making method	Bellman–Zadeh			LINMAP			TOPSIS		
	Power	η_{SE}	$\Sigma\Delta P_i$	Power	η_{SE}	$\Sigma\Delta P_i$	Power	η_{SE}	$\Sigma\Delta P_i$
Average error (%)	2.70	2.90	5.30	2.80	3.80	4.60	3.00	2.00	6.30
Max error (%)	3.80	4.60	10.50	5.00	6.60	10.40	8.50	2.60	10.70

Pareto optimal frontier for three objective functions, output power, thermal efficiency of Stirling engines and system pressure losses are represented in Fig. 4a. The chosen points based on decision-making methods are shown, too. The equation of the Pareto optimal frontier is obtained and indicated in Fig. 4b, which can be applied to gain system pressure losses, as following:

$$\Delta P_i = 1.736 \times 10^4 + 7795\eta + 4517P + 1597\eta P + 3472\eta^2 + 77.77P^2$$

$$R^2 = 98.45 \quad (44)$$

Pareto optimal frontier for output power and thermal efficiency are shown in Fig. 5, which are two objective functions. It is clear that output power is between 5300 W and 8300 W in the curve fitted diagram:

$$\eta = -2.23 \times 10^{-14}P^3 - 1.494 \times 10^{-10}P^2 - 3.917 \times 10^{-6}P + 0.204 \quad (45)$$

Pareto optimal frontier for system pressure losses and efficiency are shown in Fig. 6, which are two objective functions. It is obvious that thermal efficiency is between 0.03 and 0.18 in the curve fitted diagram:

$$\Delta P_i = 9294e^{1.497\eta} + 0.006339e^{79.15\eta} \quad (46)$$

Pareto optimal frontier for system pressure losses and output power are shown in Fig. 7, which are two objective functions. It can be seen that output power is between 500 W and 8500 W in the curve fitted diagram:

$$\Delta P_i = 1.991 \times 10^{-10}P^{3.6} + 1.026 \times 10^4 \quad (47)$$

The optimal results for decision parameters and objective functions using LINMAP, TOPSIS, Bellman–Zadeh decision-making methods are summarized in Table 1.

Maximum and mean error analyses for objective functions are categorized in Table 2. The analysis is done for all mentioned decision-making methods.

Sensitivity analysis for volumetric ratio and wire diameter are shown in Tables 3a and 3b based on the highest and lowest values. Decision variables are represented in Table 3a using TOPSIS decision-making method. The results of sensitivity analysis on objective functions are indicated in Table 3b using TOPSIS decision-making method.

Table 3a

Row data for the sensitivity analysis decision variables to variation of design parameter.

Parameter	Proposed value	Variations range		Decision variables										
				n_r [rpm]	P_m [kPa]	s [mm]	T_H [K]	T_L [K]	ΔT_H [K]	ΔT_L [K]	N_R	L [mm]	D_C [mm]	D_R [mm]
λ	1.2	Lower	1.2	2120	2550.3	60.5	989.6	298.4	74.4	11.8	339	70.0	101.6	59.5
		Upper	1.5	2769	1778.5	60.7	981.3	289.9	67.2	6.9	297	67.8	83.5	56.1
$d(\text{m})$	4×10^{-5}	Lower	3×10^{-5}	2193	2485.7	60.1	991.2	296.6	73.3	9.1	306	72.6	101.5	59.9
		Upper	8×10^{-5}	1994	2210.3	60.4	984.9	292.3	67.5	7.1	278	71.8	93.6	59.3

Note: Sensitivity analysis performed according to TOPSIS method.

Table 3b

Row data for the sensitivity analysis of objectives to variation of design parameter.

Parameters	Proposed value	Variations range		Objective functions		
				Power [kW]	η_{SE} [%]	$\Sigma \Delta P_i$ [kPa]
λ	1.2	Lower	1.2	6075.8	14.56	19.689
		Upper	1.5	20897.9	26.86	27.053
d (m)	4×10^{-5}	Lower	3×10^{-5}	6287.3	14.39	19.838
		Upper	8×10^{-5}	5842.2	14.74	17.710

Note: Sensitivity analysis performed according to TOPSIS method.

7. Conclusions

With regard to previous investigations on Stirling cycle and Stirling engine, finite speed thermodynamic has more acceptable results in comparison to the others due to considering external irreversibilities. The effects of heat source temperature, temperature difference between heat source and working fluid on the heat side, engine rotation speed, mean effective pressure, stroke and etc on output power, thermal efficiency and system pressure losses are scrutinized utilizing NSGAI algorithm. The consequences can be implemented for designing and surveying Stirling engine performance and robustness.

Nomenclature

b	coefficient $\in (0,2)$
C_v	constant volume specific heat
Cl_i	decision index in TOPSIS method
d	deviation index
d_{i+}	deviation or distance of i th solution from ideal solution
d_{i-}	deviation or distance of i th solution from non-ideal solution
D	diameter (m)
d	wire diameter (m)
f	coefficient related to the friction contribution $\in (0,1)$
h	heat transfer coefficient ($\text{W m}^{-2} \text{K}^{-1}$)
m	mass of the gas
N	number of gauzes of the matrix, number of regenerators per cylinder
n_r	rotation speed
p	pressure
Δp	pressure loss
P	output power
$\dot{Q}$	heat transfer rate
R	gas constant
s	stroke
T	temperature
ΔT	temperature difference
i	i th objective
j	j th solution
X	vector of decision variables, regenerative losses coefficient

Greek letter

λ	ratio of volume during the regenerative processes (volumetric ratio)
ϵ_R	effectiveness of the regenerator
η	efficiency
γ	specific heat ratio
μ	fuzzy membership function
ϵ	emissivity factor
δ	the Stefan's constant
ν	viscosity of the working gas ($\text{m}^2 \text{s}^{-1}$)

Subscripts

c	cylinder, related to the Carnot cycle
f	friction
H	heat source
L	heat sink
SE	Stirling engine
II	related to the second law
$1-4$	the processes states
g	gas
$aver$	average
m	mean, the system
app	collector aperture
$thrott$	throttling

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